History of Mathematics Project: Learning Journeys for Kids and Others

Bernat Espigule, Wolfram Research

#WolframTechConf

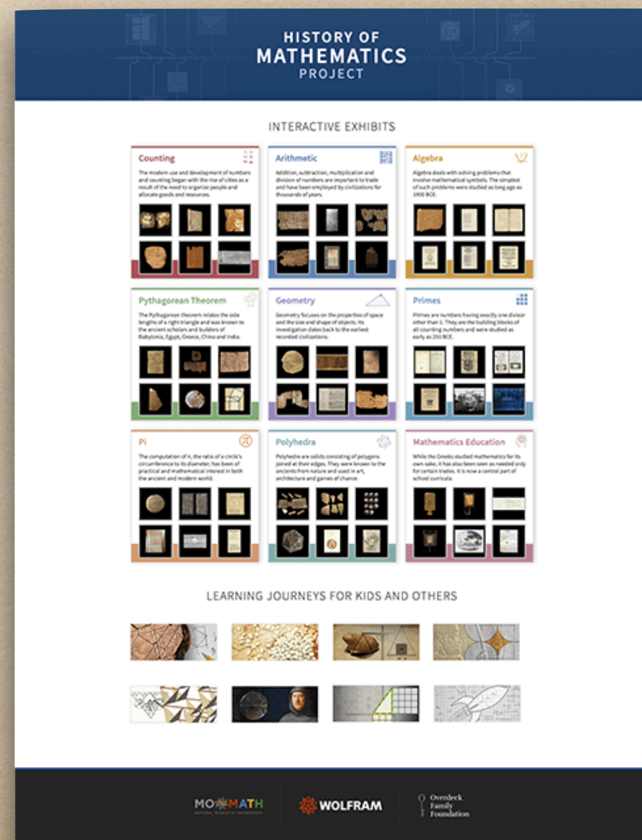
@bernatree

Website Organization

- Home page (history-of-mathematics.org)
- 9 virtual exhibits
- 74 artifact pages
- 8 learning journeys

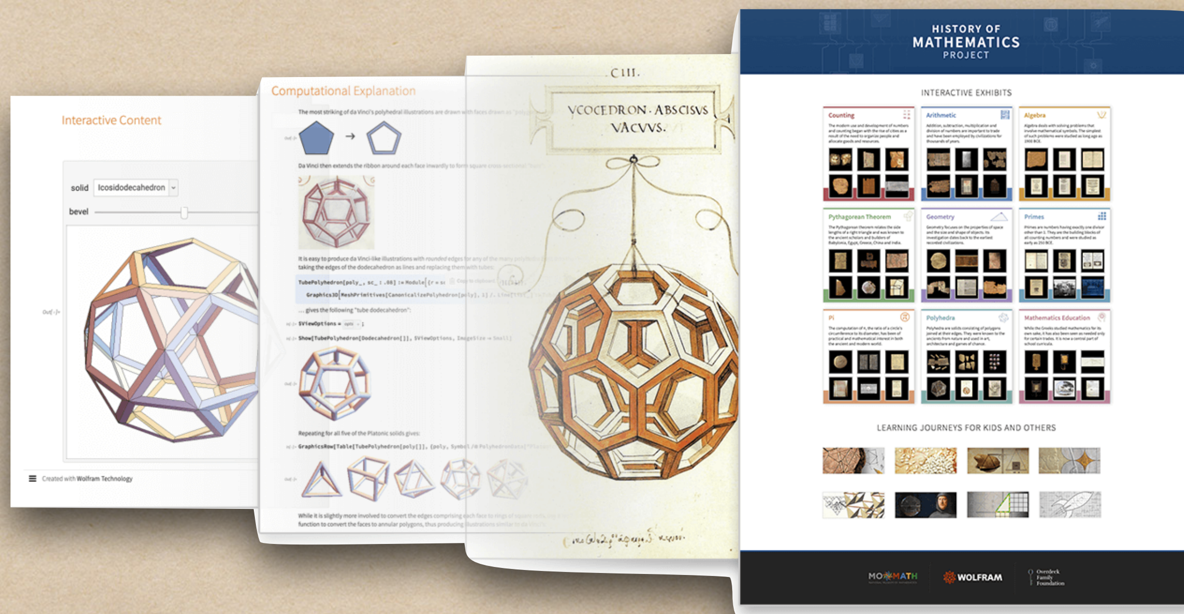
Ancient Games of Chance

Balancing Ducks, Frogs and Grasshoppers



The History of Mathematics Development Team

- Andrea Gerlach
- Eric Weisstein
- Bernat Espigulé
- Sarah Keim Williams
- Lori Goodman



- with additional contributions from 50+ domain experts in relevant areas of the history of mathematics, notation, and the study of antiquities

9 Interactive Exhibits + 8 Learning Journeys

Counting

The modern use and development of numbers and counting began with the rise of cities as a result of the need to organize people and allocate goods and resources.

Arithmetic

Addition, subtraction, multiplication and division of numbers are important to trade and have been employed by civilizations for thousands of years.

Algebra

Algebra deals with solving problems that involve mathematical symbols. The simplest of such problems were studied as long ago as 1900 BCE.

Pythagorean Theorem

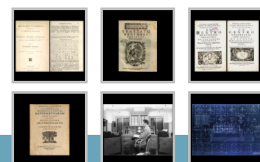
The Pythagorean theorem relates the side lengths of a right triangle and was known to the ancient scholars and builders of Babylonia, Egypt, Greece, China and India.

Geometry

Geometry focuses on the properties of space and the size and shape of objects. Its investigation dates back to the earliest recorded civilizations.

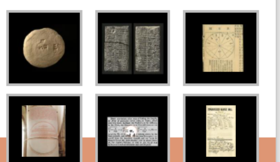
Primes

Primes are numbers having exactly one divisor other than 1. They are the building blocks of all counting numbers and were studied as early as 250 BCE.



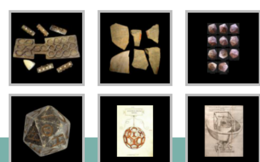
Pi

The computation of π, the ratio of a circle's circumference to its diameter, has been of practical and mathematical interest in both the ancient and modern world.



Polyhedra

Polyhedra are solids consisting of polygons joined at their edges. They were known to the ancients from nature and used in art, architecture and games of chance.



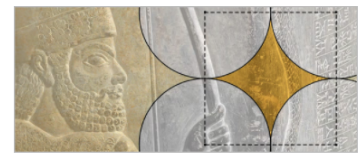
Mathematics Education

While the Greeks studied mathematics for its own sake, it has also been seen as needed only for certain trades. It is now a central part of school curricula.

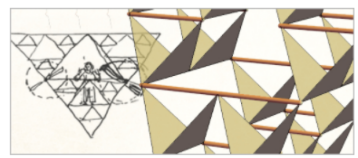
Mathematical Beans and Knotted Strings

Balancing Ducks, Frogs and Grasshoppers

Show Your Work!

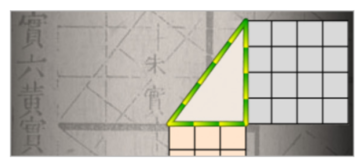


Squaring the Apsamikku Circle

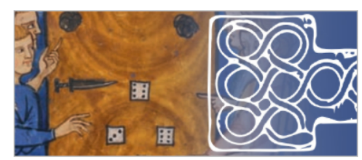


Making Machines Fly

The Mathematics of a Masterpiece

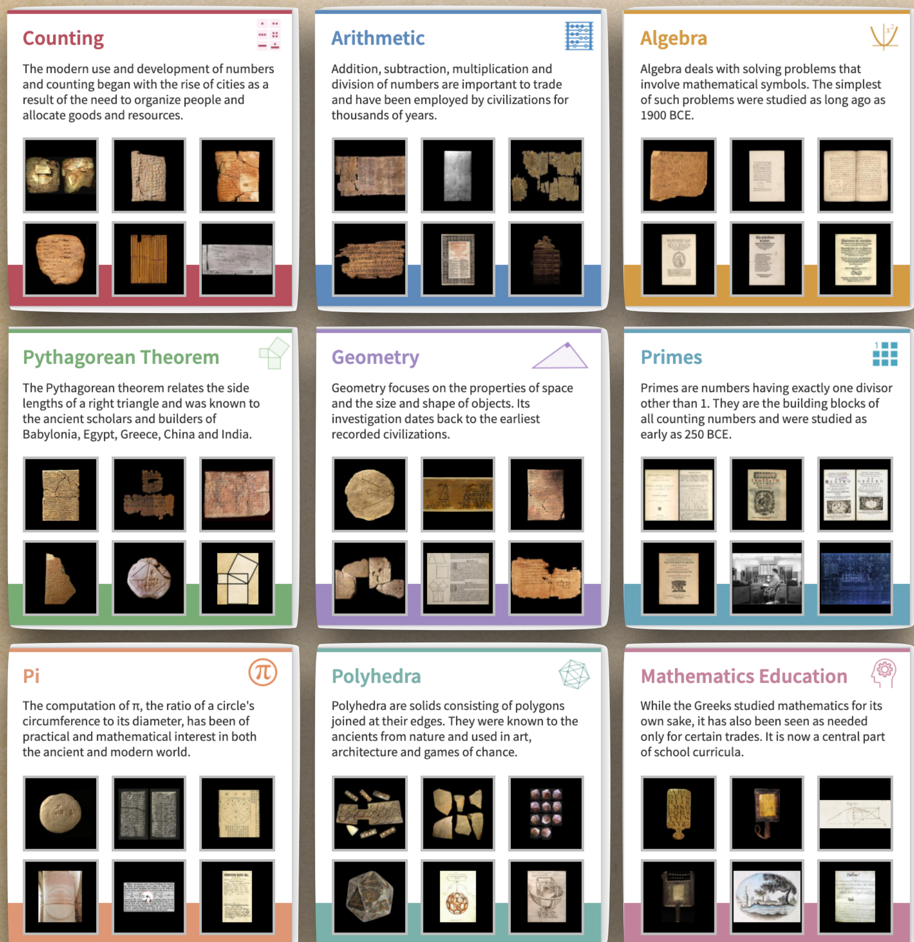


Ancient Right Triangles



Ancient Games of Chance

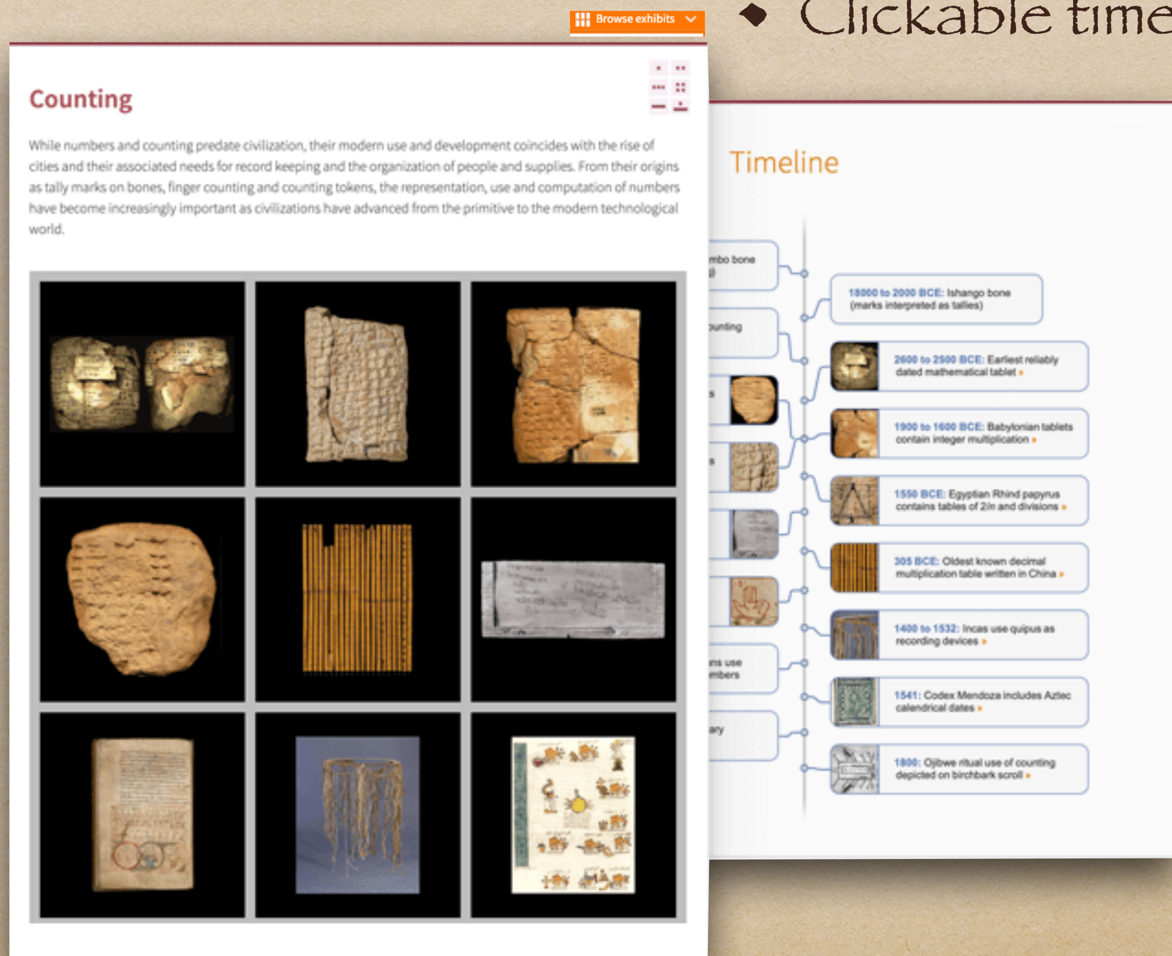
9 Interactive Exhibits



- Counting
- Arithmetic
- Algebra
- Pythagorean Theorem
- Geometry
- Primes
- Pi
- Polyhedra
- Mathematics Education

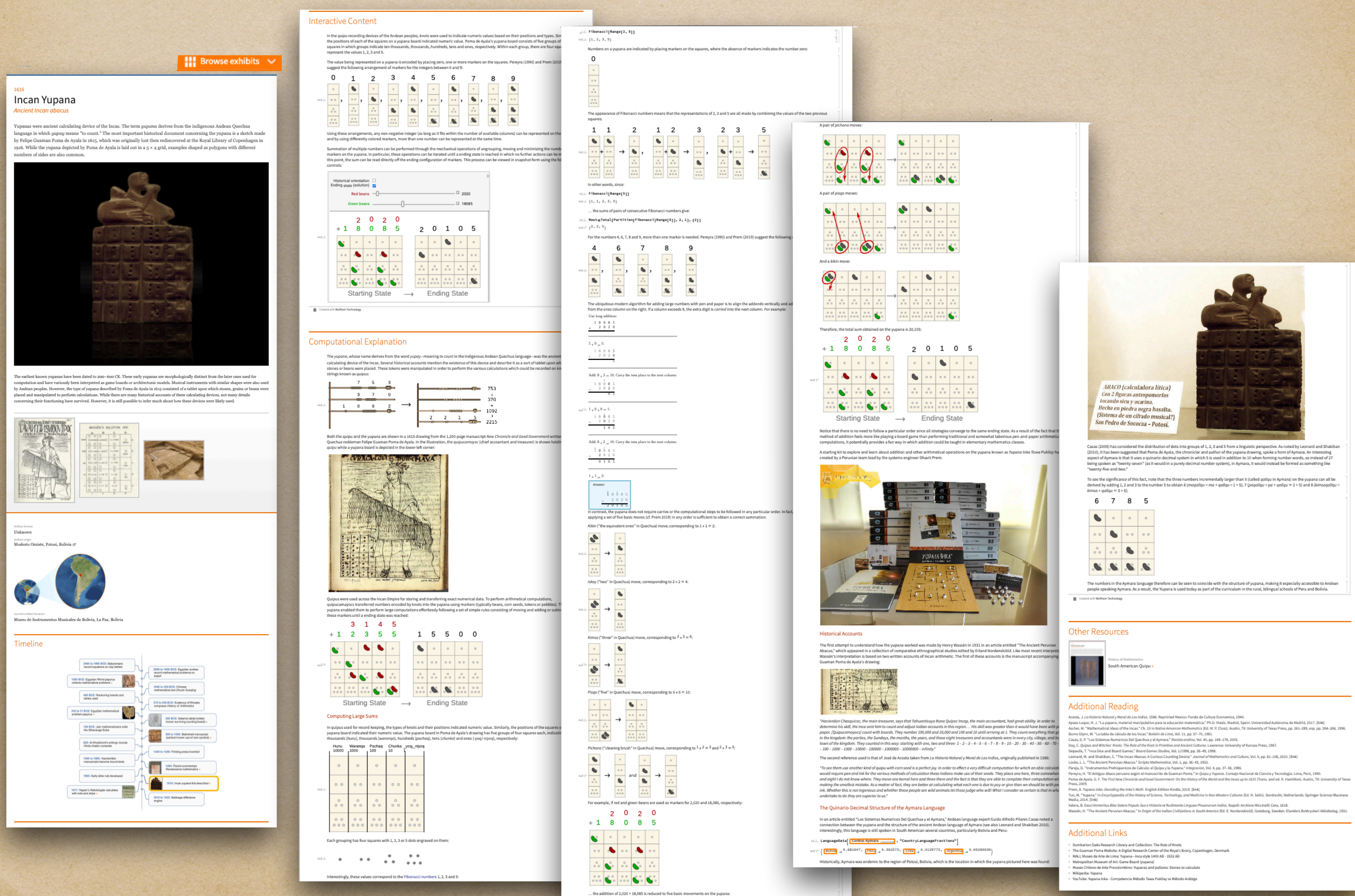
Each Interactive Exhibit Page Contains:

- Navigation to other exhibits
- Short description
- Clickable timeline



- Clickable thumbnails for several artifacts

Each Math Artifact Page Contains:



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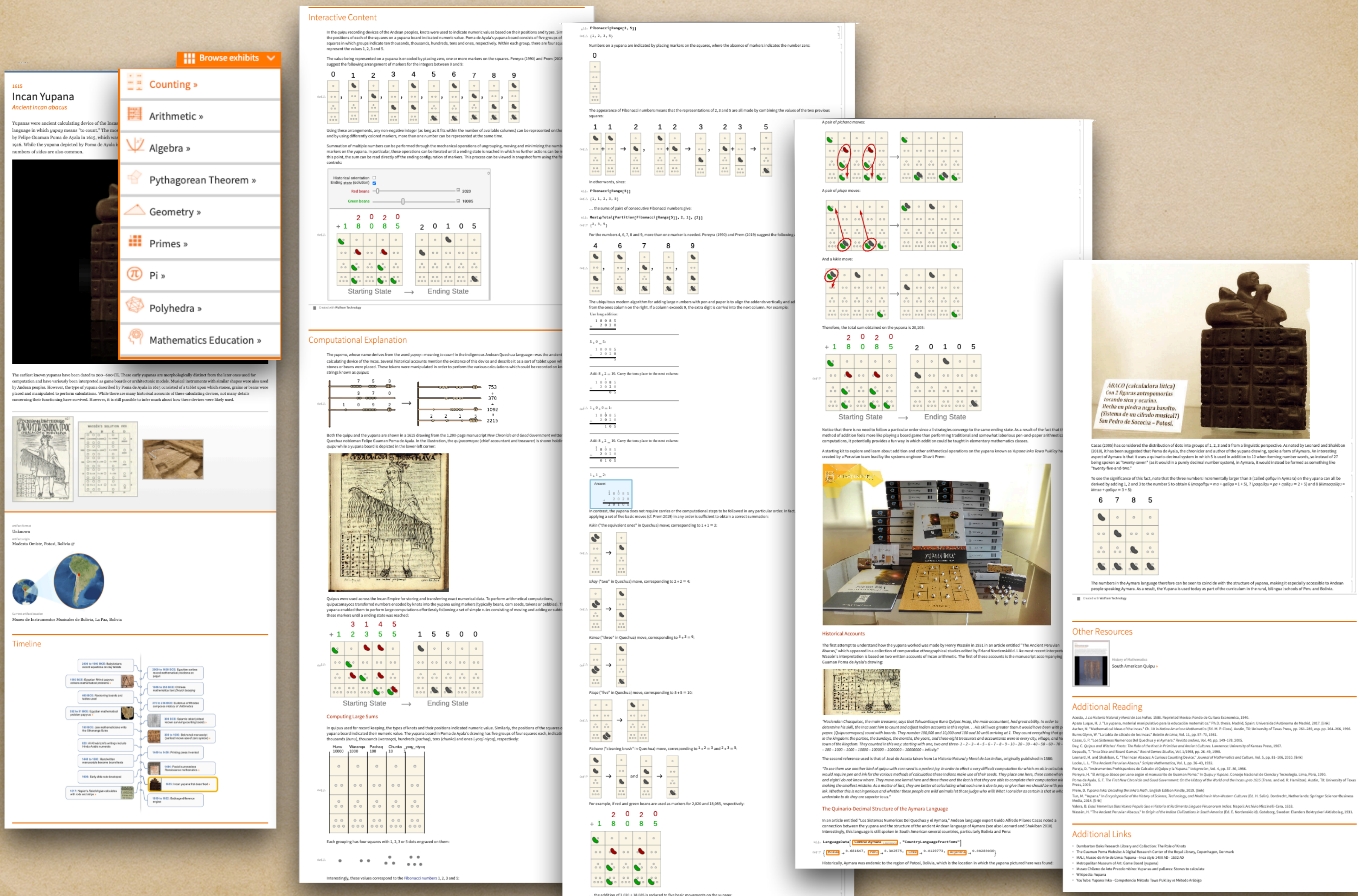

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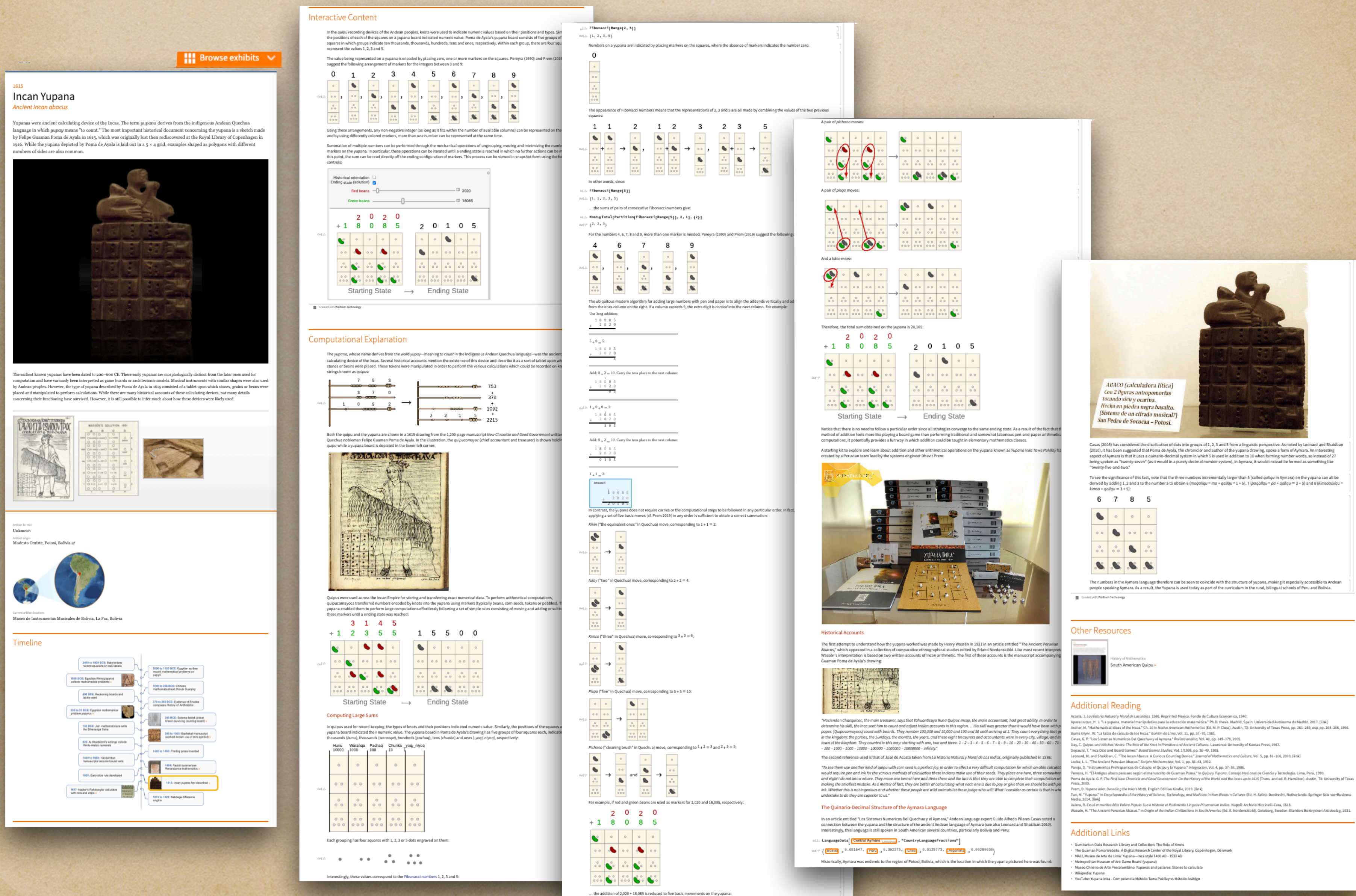

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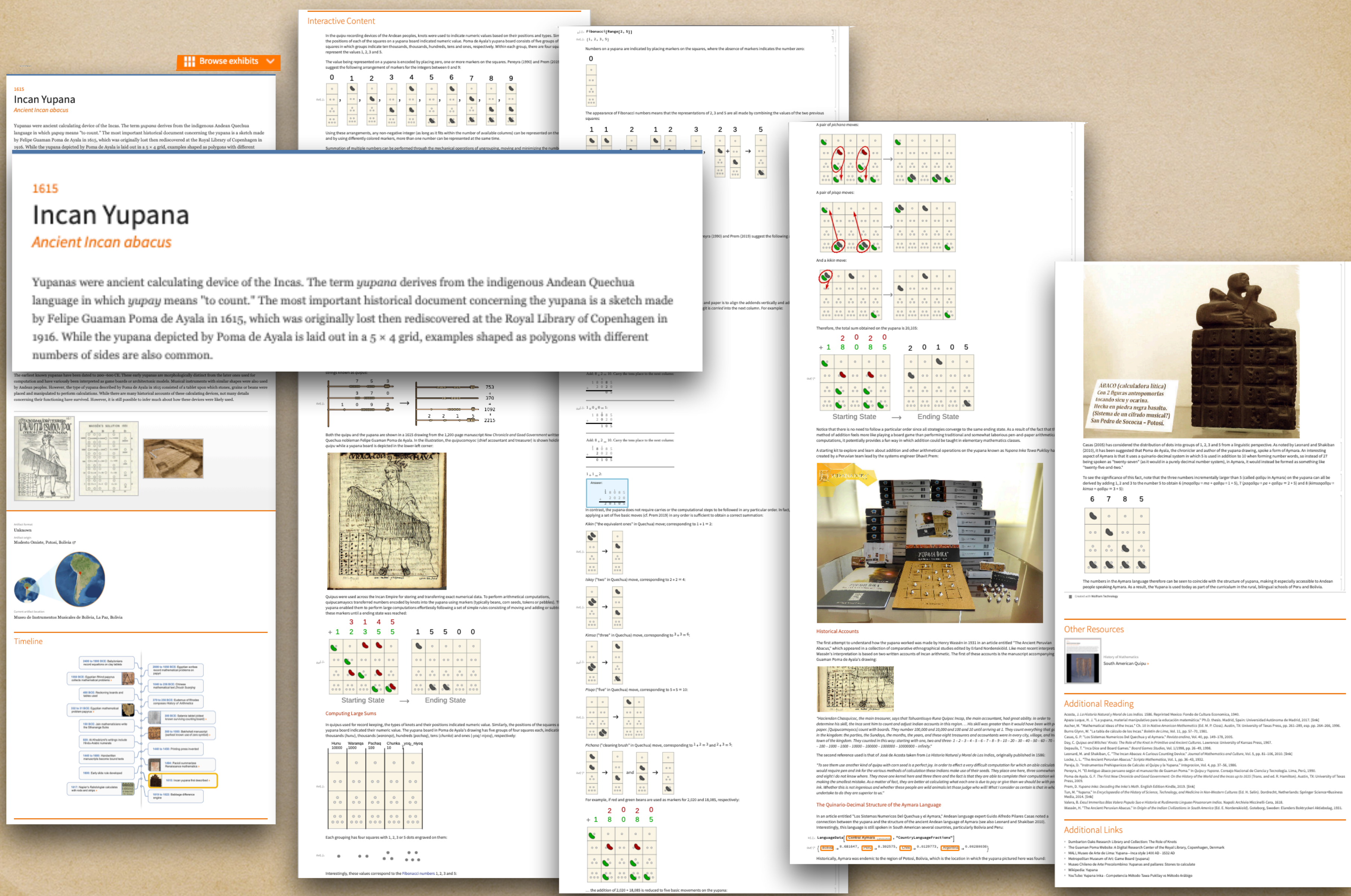

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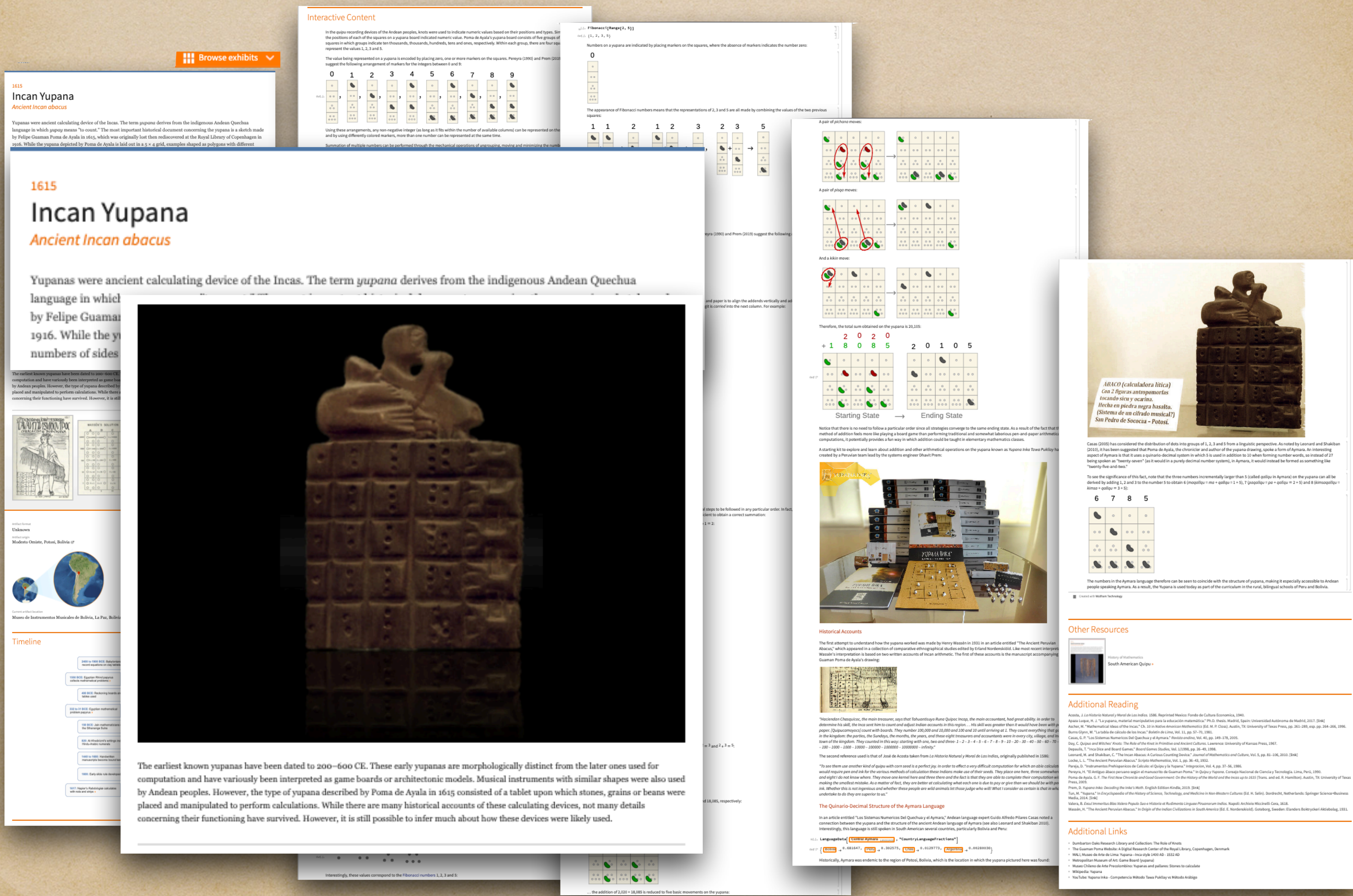

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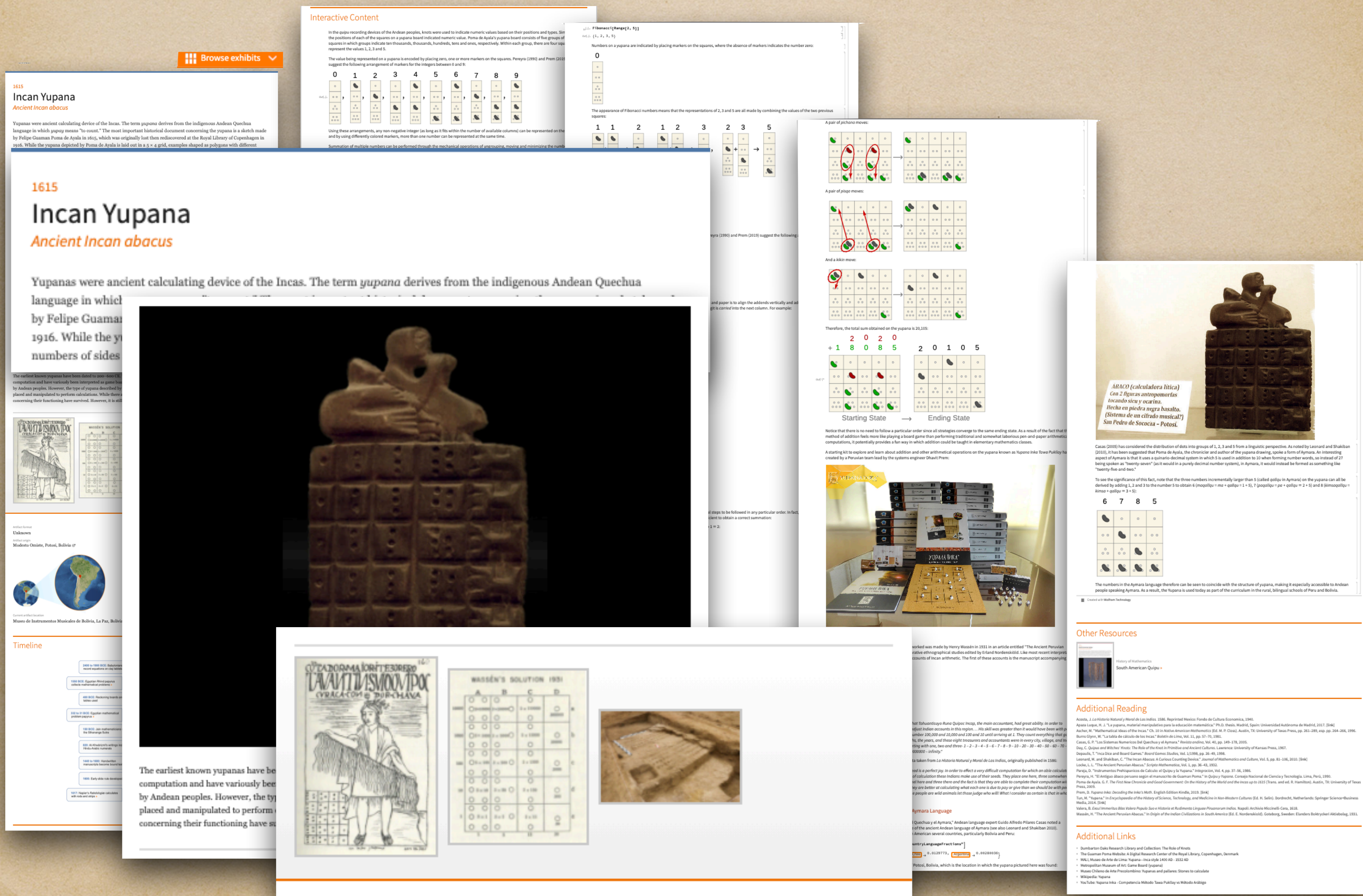

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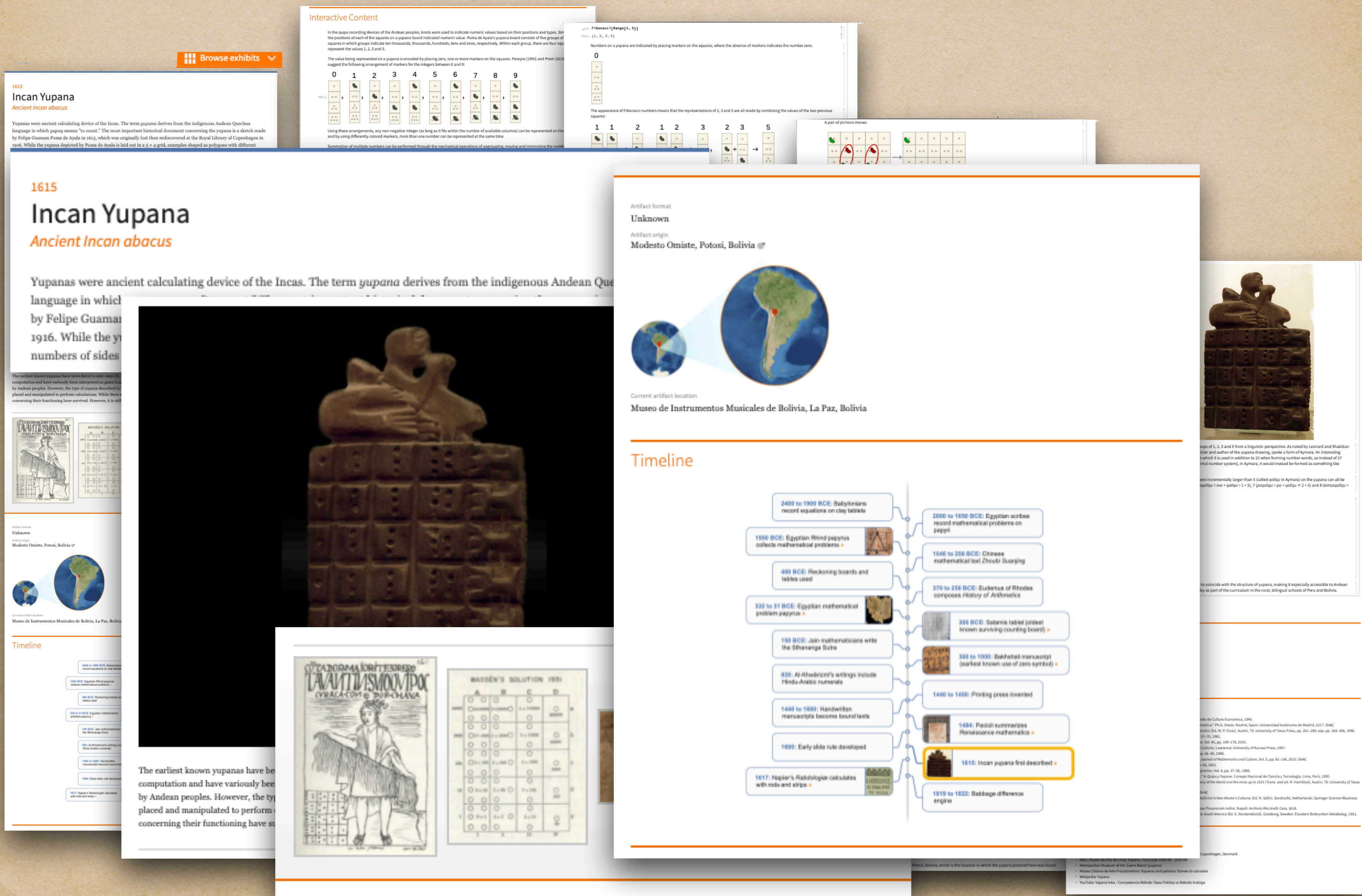

Interactive and Computational Content

- Interactive content gives a Manipulate-based exploration of artifact content including some basic background and information
- Computational explanations give detailed explanations of the mathematical content of the artifact that make extensive use of the Wolfram Language

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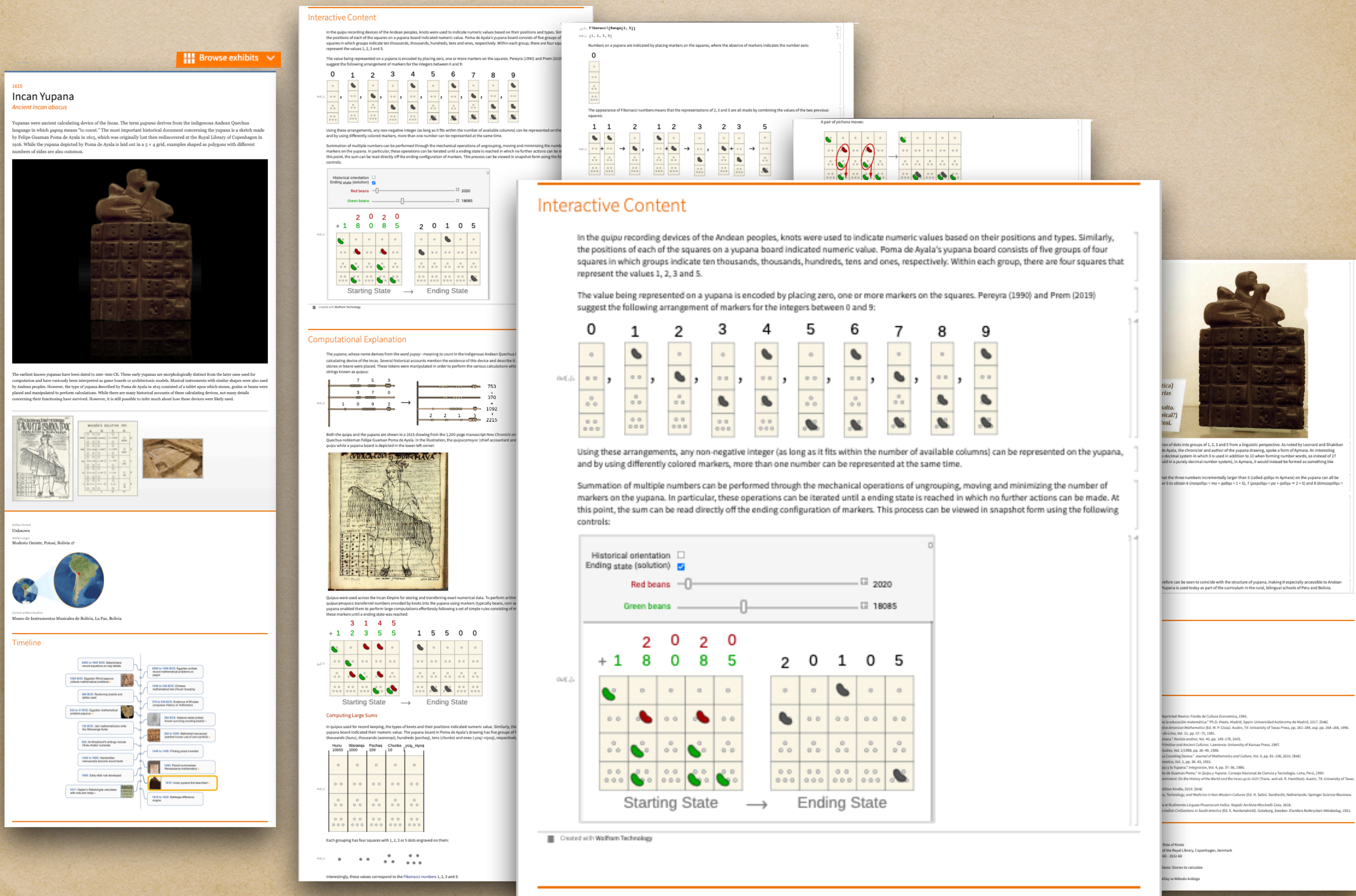

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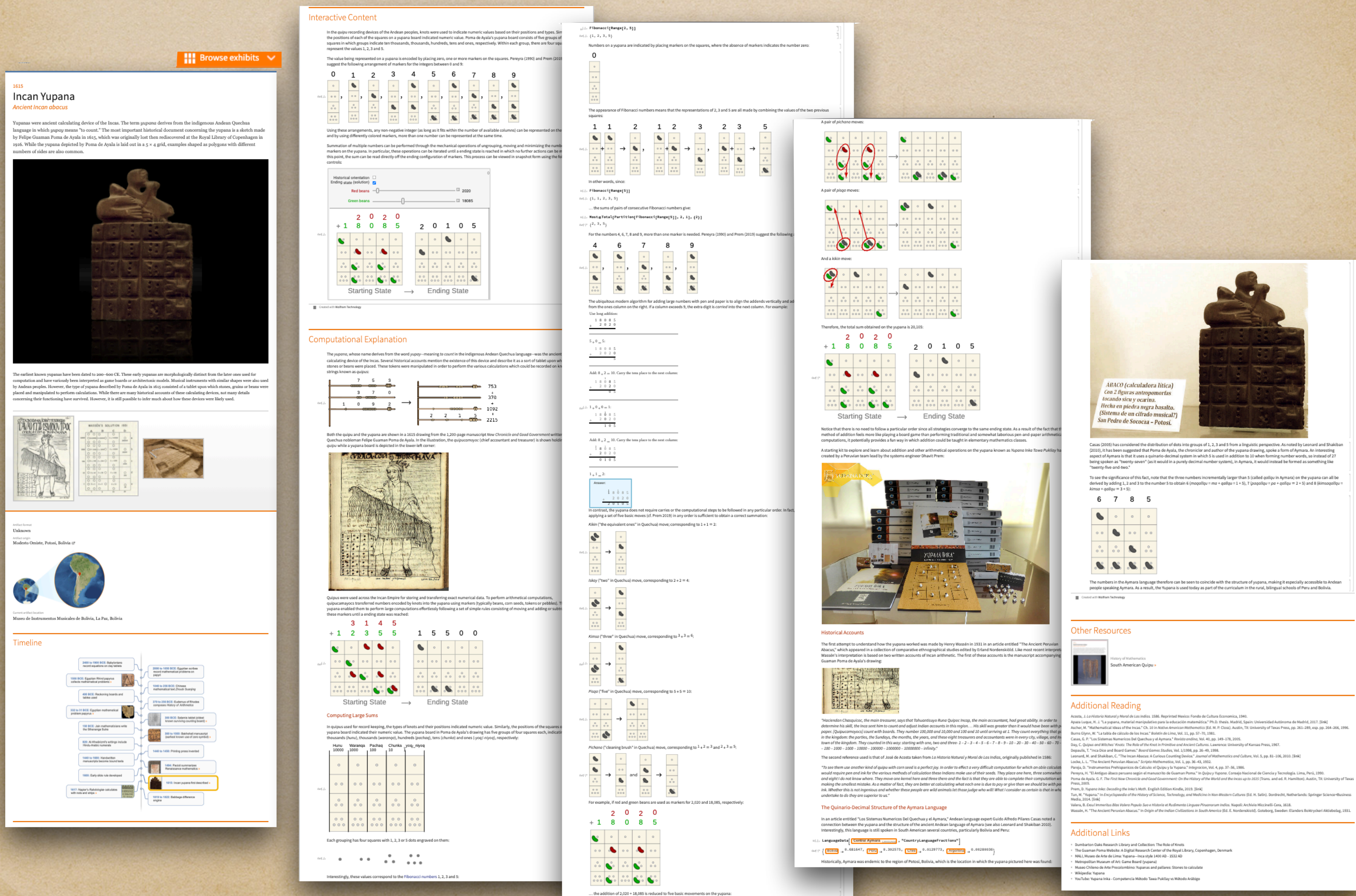

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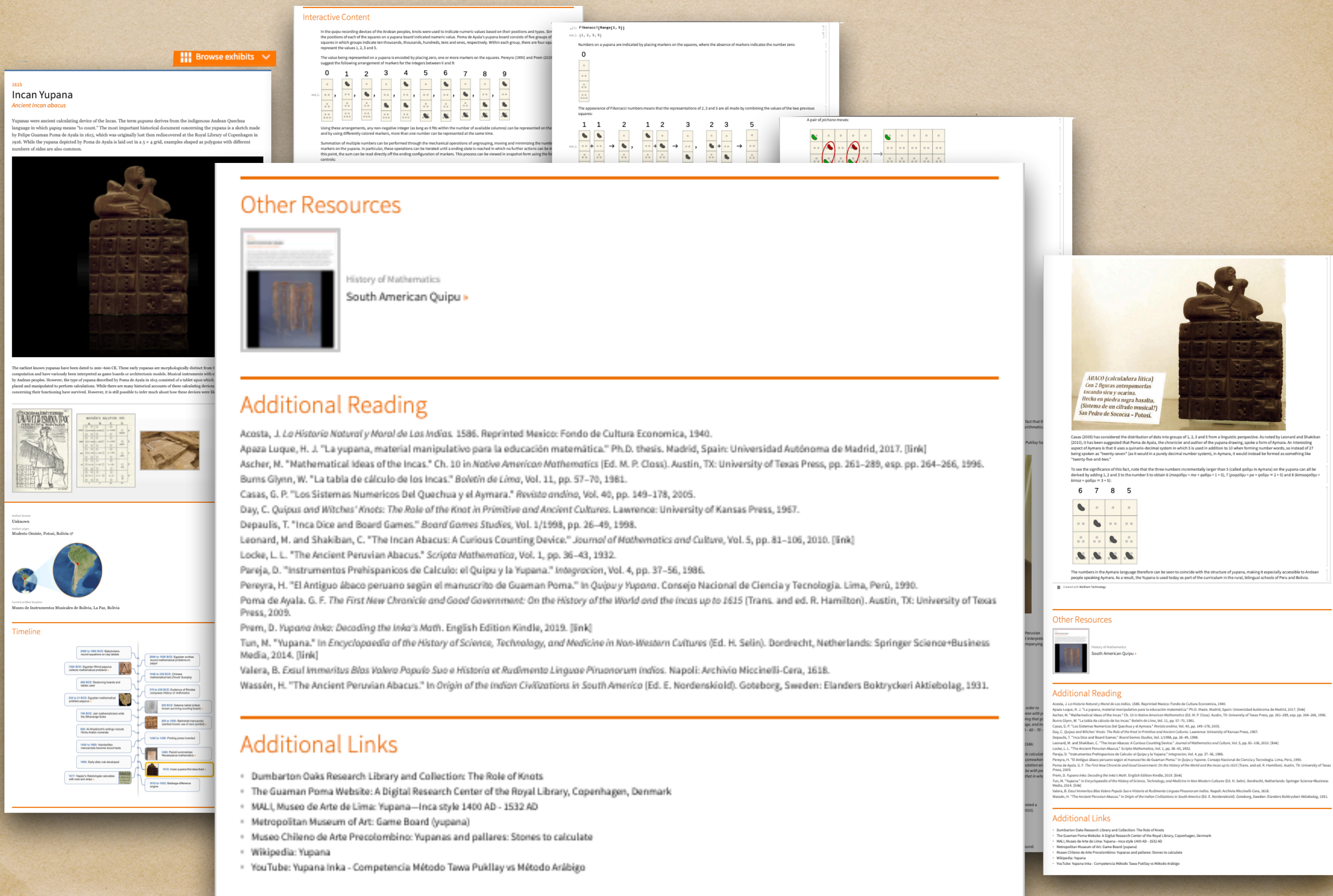
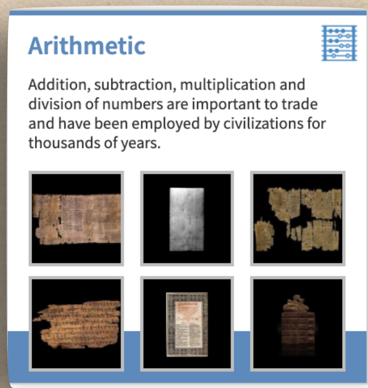
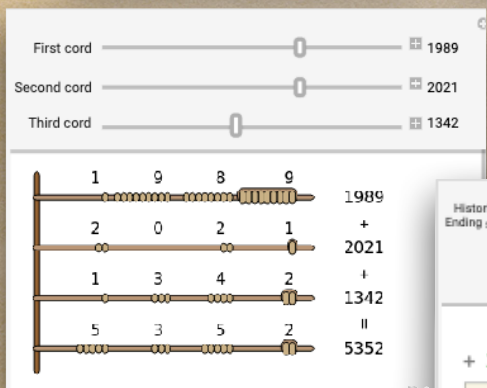


8 Learning Journeys





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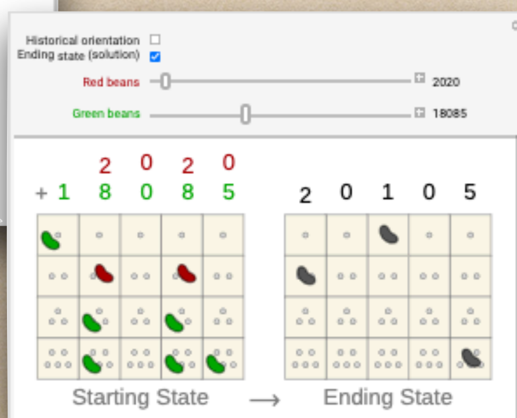
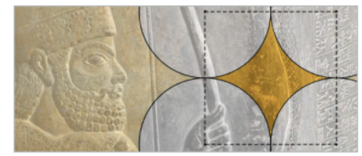


Mathematical Beans and Knotted Strings

Balancing Ducks, Frogs and Grasshoppers

Show Your Work!



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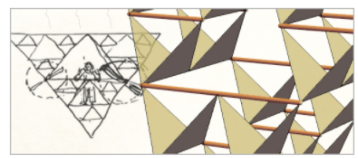

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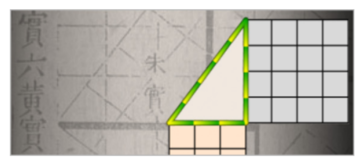
The Mathematics of a Masterpiece



Ancient Right Triangles

Ancient Games of Chance

Learning Journeys

- Intended as an engaging and fun "journey" through mathematical artifacts
- Aimed at students and other virtual museum visitors who are interested in the "mathematical story"
- Useful for classroom exploration or as a teaching tool
- Contain images and links to individual artifacts
- Include interactive content
- Primarily visual and descriptive with minimal mathematics
- 8 learning journeys

Learning Journeys

8 Learning Journeys

- Mathematical Beans and Knotted Strings
- Balancing Ducks, Frogs and Grasshoppers
- Show Your Work!
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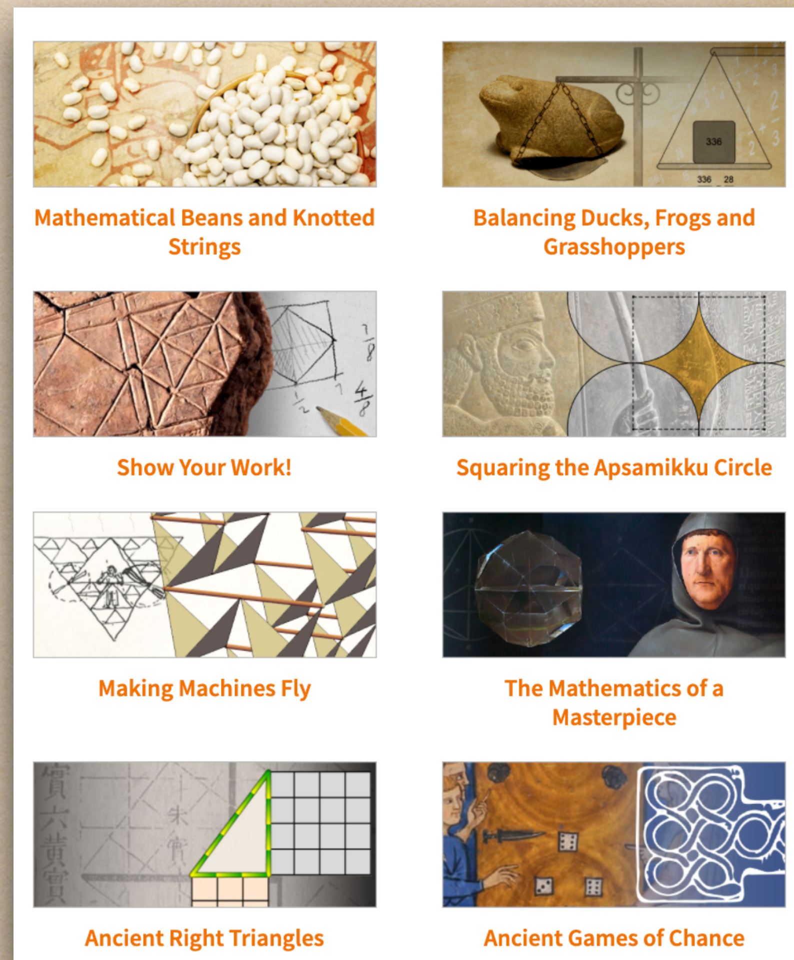

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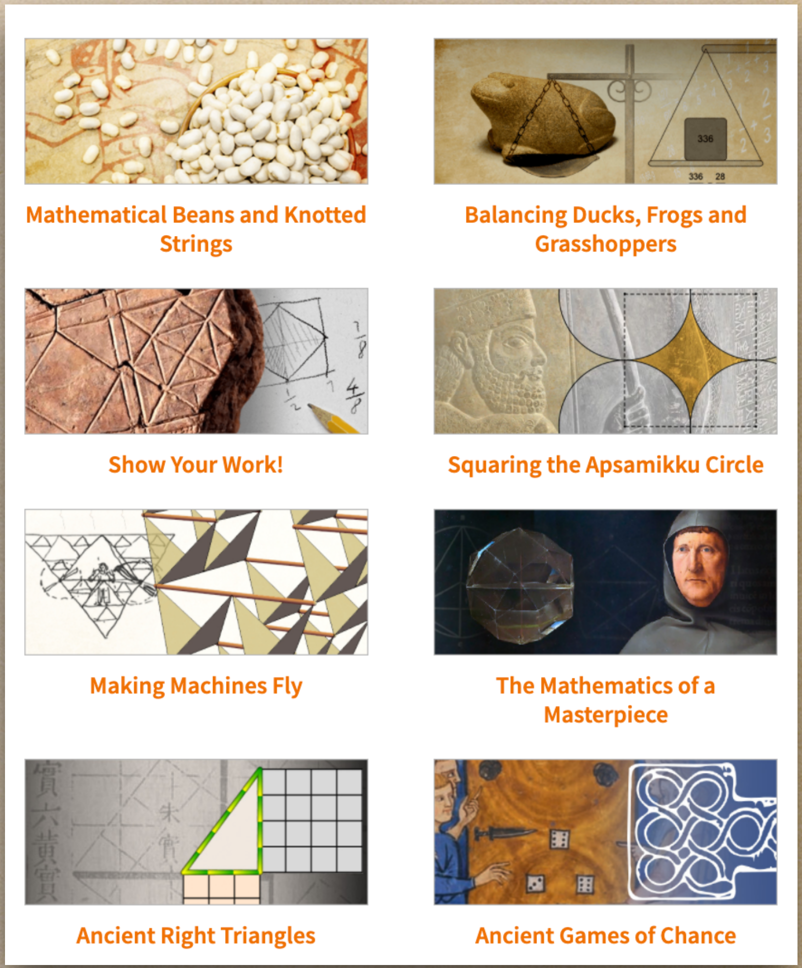

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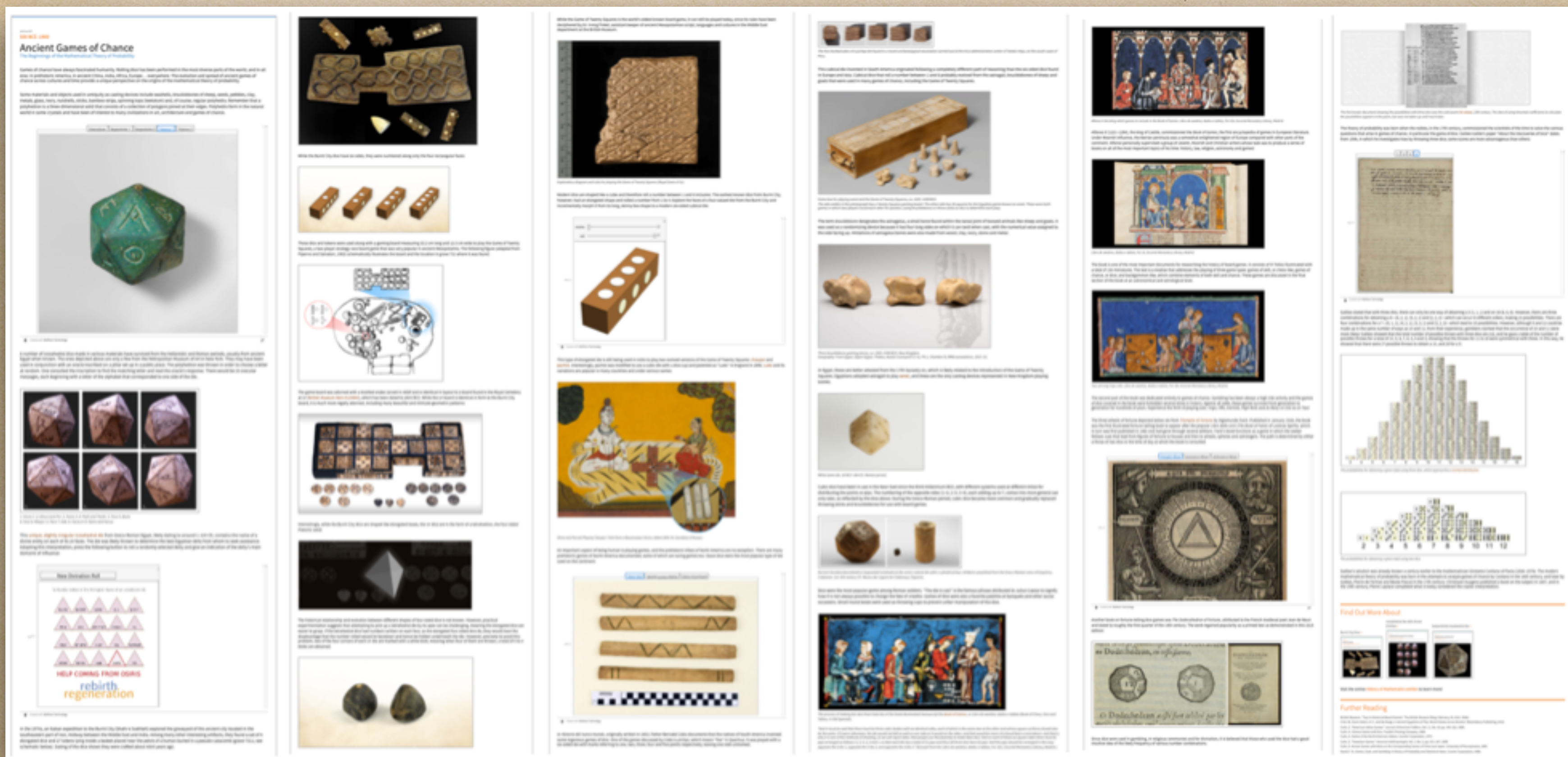


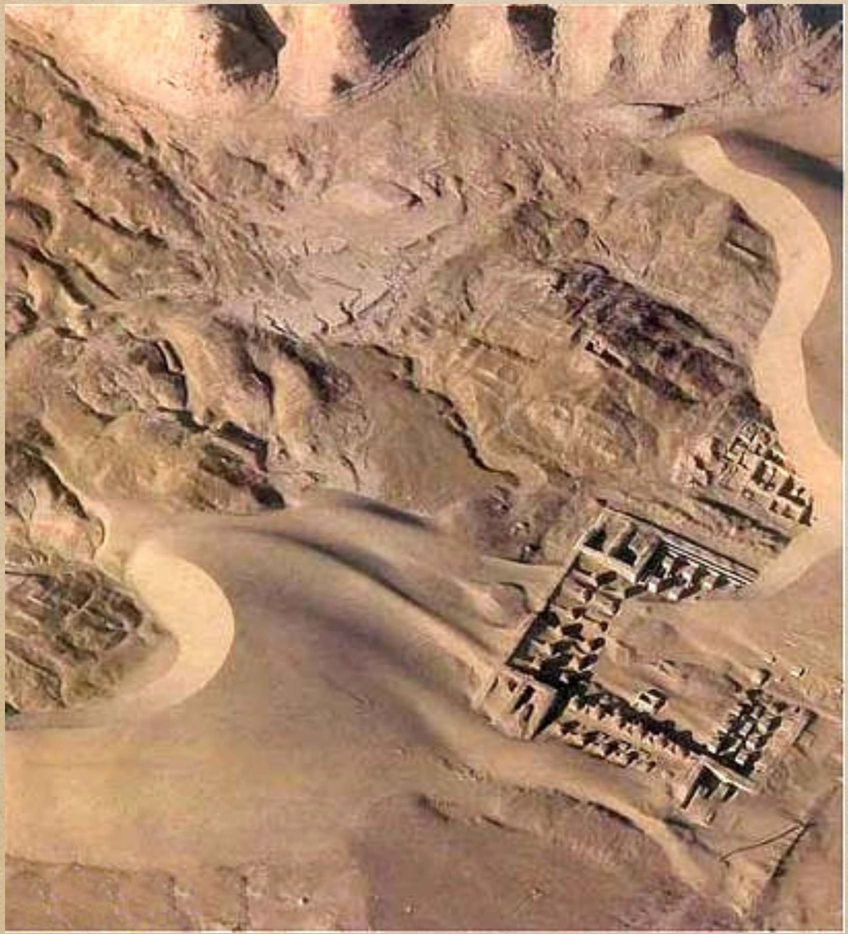
Ancient Games of Chance

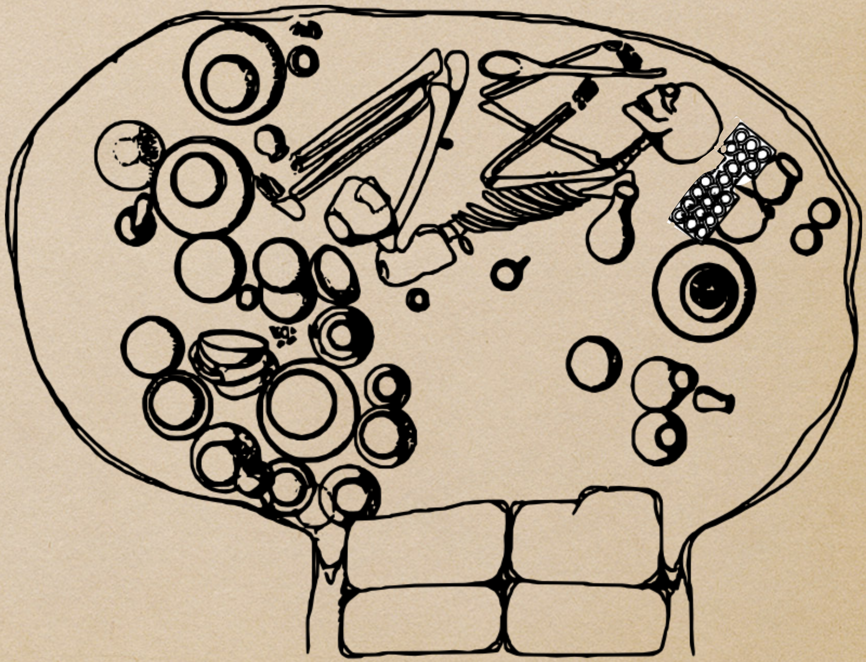
Shahr-e Sūkhté (Persian: شهر سوخته, meaning "The Burnt City")

Dice crafted about 4500 years ago and discovered in the 1970s by an Italian expedition to the Burnt City ruins located in nowadays Iran, midway between the Middle East and the Indus Valley, India.

Ancient Games of Chance

Shahr-e Sūkhté (Persian: شهر سوخته, meaning "The Burnt City")





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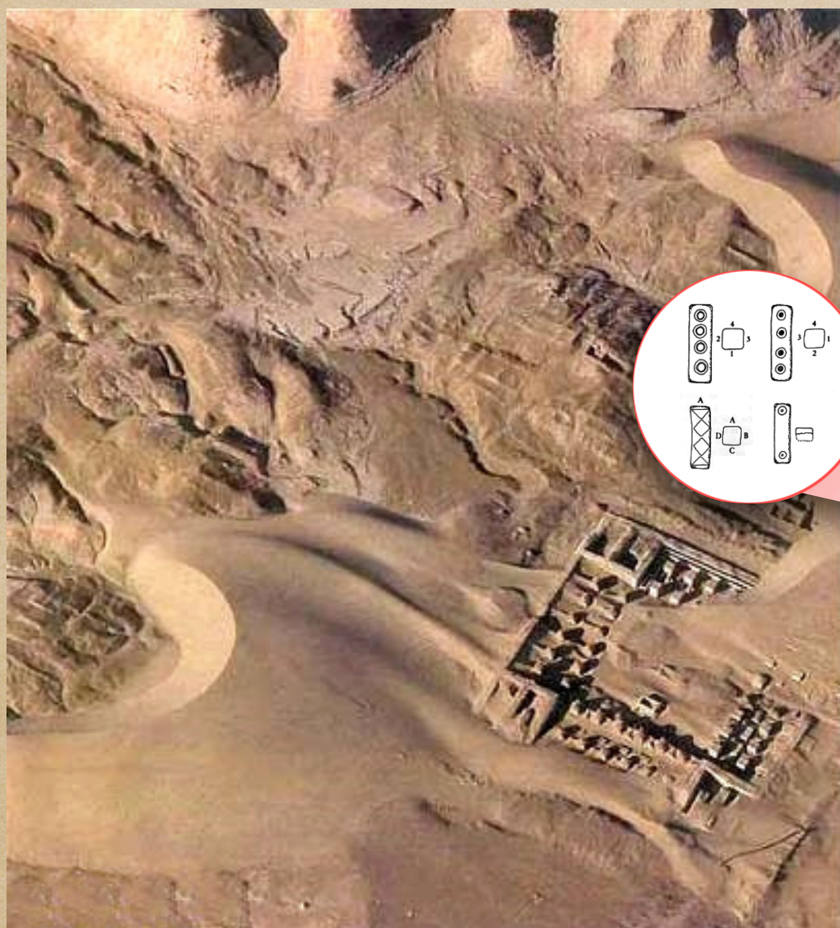

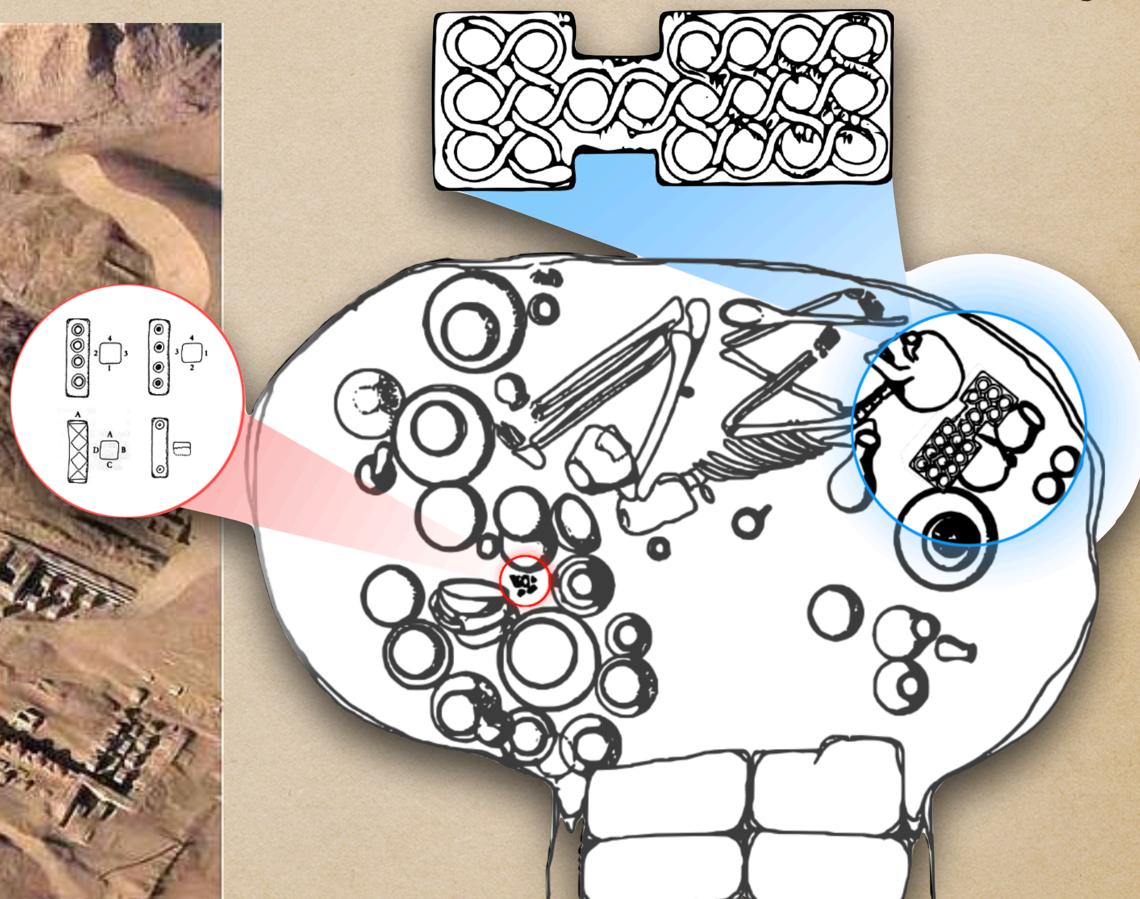

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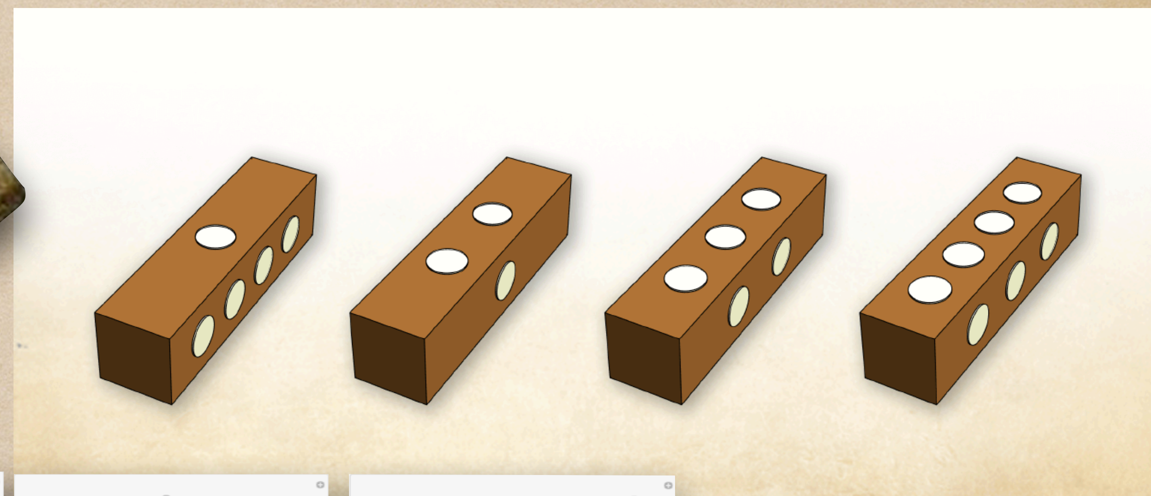
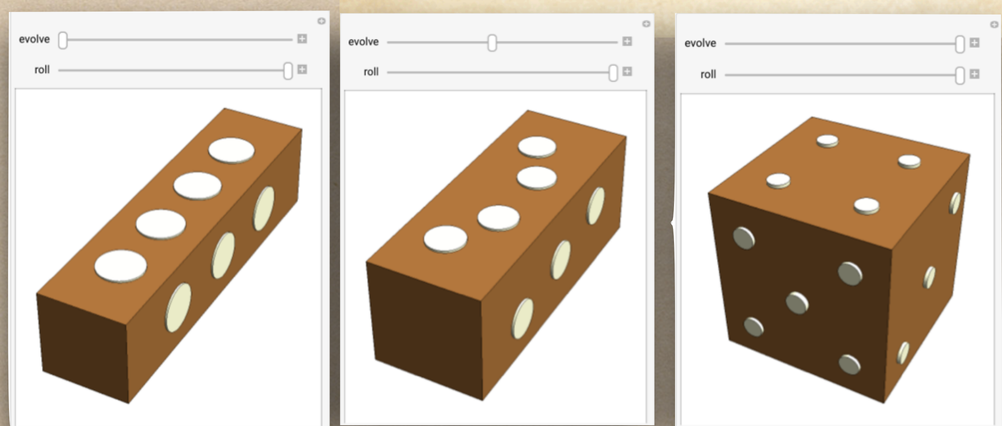




Four-valued die from the Burnt City
morphed to a modern six-sided cubical die.

Ancient Games of Chance

Shahr-e Sūkhté (Persian: شهر سوخته, meaning "The Burnt City")

The game board was adorned with a knotted snake carved in relief and is identical in layout to a board found in the Royal Cemetery at Ur.

Ancient Games of Chance

Royal Cemetery at Ur (British Museum), 2600 BCE

Game board from the Royal Cemetery at Ur (British Museum item #120840), which has been dated to 2600 BCE. While the Ur board is identical in form to the Burnt City board, it is much more regally adorned, including many beautiful and intricate geometric patterns.

Ancient Games of Chance

Royal Cemetery at Ur (British Museum), 2600 BCE

While the Game of Twenty Squares is the world's oldest known board game, it can still be played today, since its rules have been deciphered by Dr. Irving Finkel, assistant keeper of ancient Mesopotamian script, languages and cultures in the Middle East department at the British Museum.

Ancient Games of Chance

Egyptian game box, ca. 1635–1458 BCE.

Sheep knucklebones were used as a randomizing device because it has four long sides on which it can land when cast, with the numerical value assigned to the side facing up.

Ancient Games of Chance

Egyptian game box, ca. 1635–1458 BCE.

Sheep knucklebones were used as a randomizing device because it has four long sides on which it can land when cast, with the numerical value assigned to the side facing up.

White stone die. 30 BCE–364 CE. Roman period.

Ancient Games of Chance

The process of making fair dice from Folio 65v of the hand-illuminated manuscript the Book of Games, or Libro de axedrez, dados e tablas (Book of Chess, Dice and Tables, in Old Spanish).

During the Greco-Roman period, cubic dice became more common and gradually replaced throwing sticks and knucklebones for use with board games.

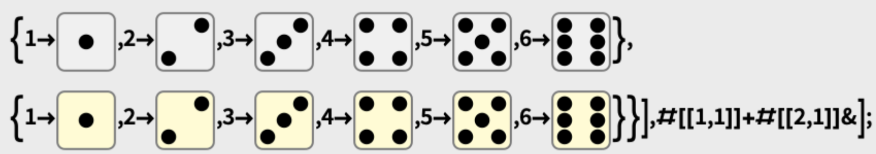
Ancient Games of Chance

Two winning triga rolls. Libro de axedrez, dados e tablas, Fol. 66r.

Ancient Games of Chance

The theory of probability was born when the nobles, in the 17th century, commissioned the scientists of the time to solve the various questions that arise in games of chance, in particular the game of dice. Galileo Galilei's paper "About the Discoveries of Dice" dates from 1596, in which he investigates how by throwing three dice, some scores are more advantageous than others.

```
probabilities=GroupBy[Tuples[{
```

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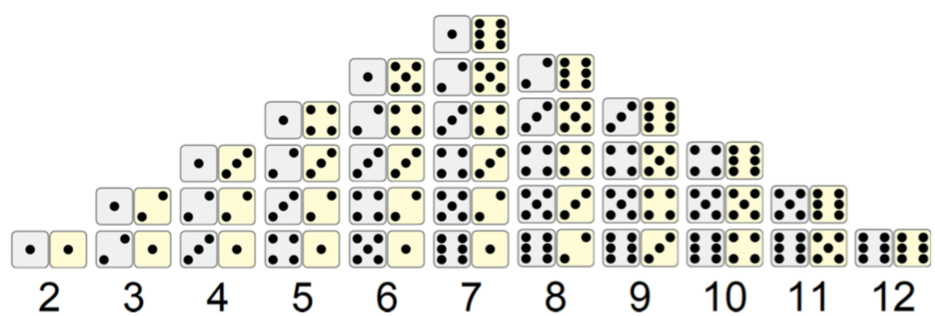
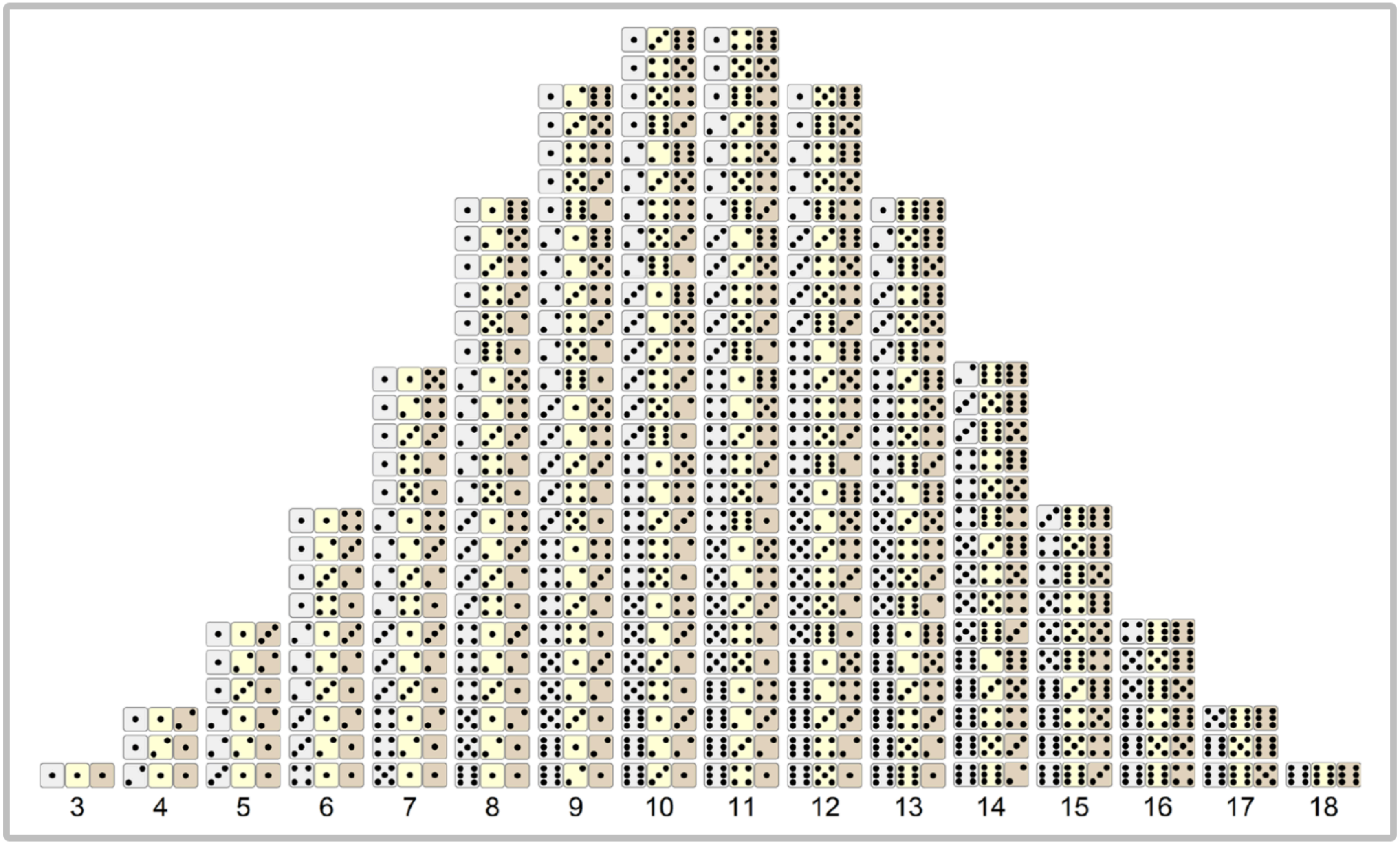


The probabilities for obtaining a given total using two dice.

Ancient Games of Chance

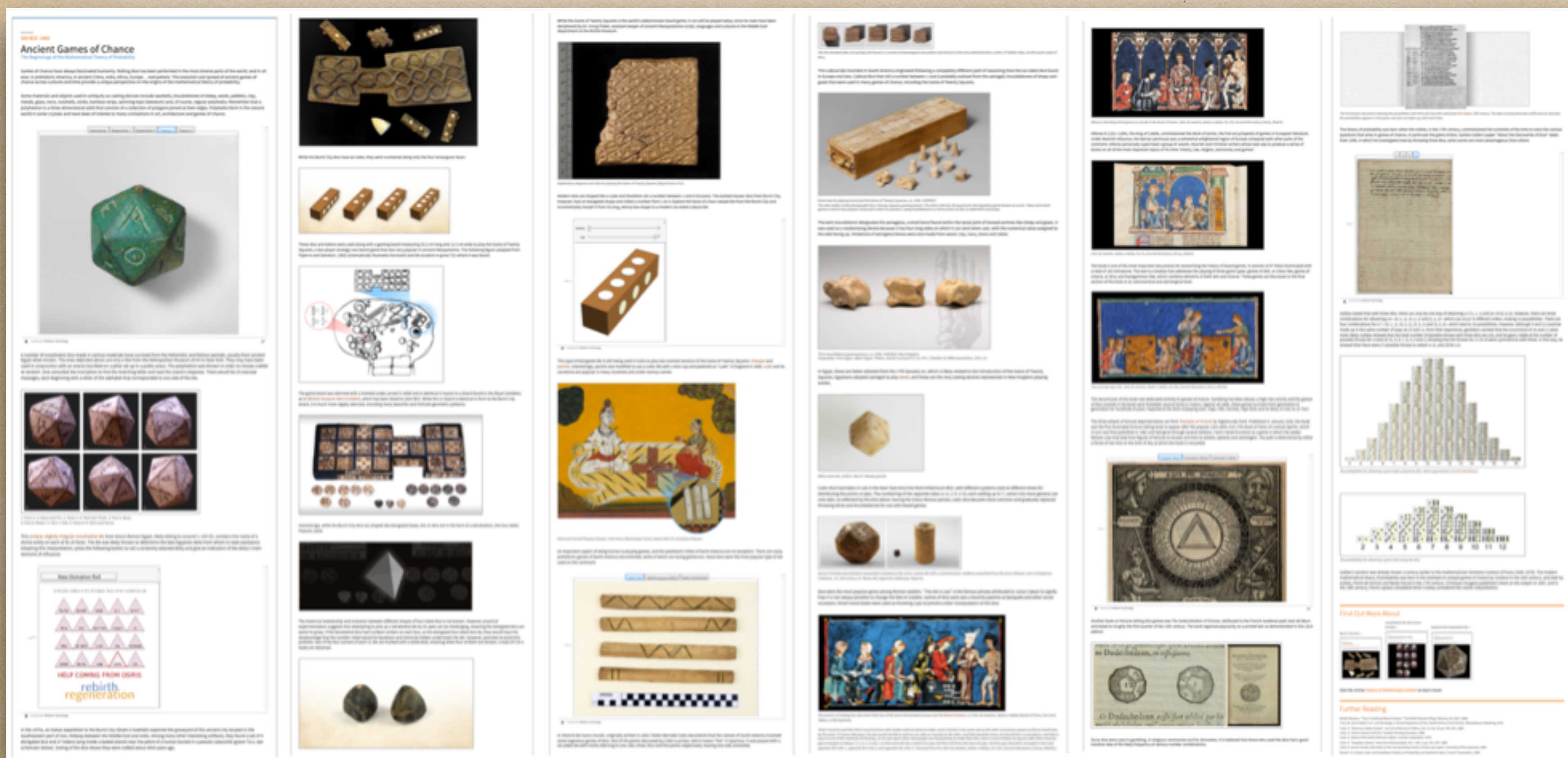


The probabilities for obtaining a given total using three dice, which approaches a normal distribution.

Ancient Games of Chance

Galileo stated that with three dice, there can only be one way of obtaining a 3 (1, 1, 1) and an 18 (6, 6, 6). However, there are three combinations for obtaining a 6—(4, 2, 1), (3, 2, 1) and (2, 2, 2)—which can occur in different orders, making 10 possibilities. There are four combinations for a 7—(5, 1, 1), (4, 2, 1), (3, 3, 1) and (3, 2, 2)—which lead to 15 possibilities. However, although 9 and 12 could be made up in the same number of ways as 10 and 11, from their experience, gamblers claimed that the occurrence of 10 and 11 were more likely! Galileo showed that the total number of possible throws with three dice are 216, and he gave a table of the number of possible throws for a total of 10, 9, 8, 7, 6, 5, 4 and 3, showing that the throws for 11 to 18 were symmetrical with these. In this way, he showed that there were 27 possible throws to obtain a 10, and 25 for a 9.

Ancient Games of Chance



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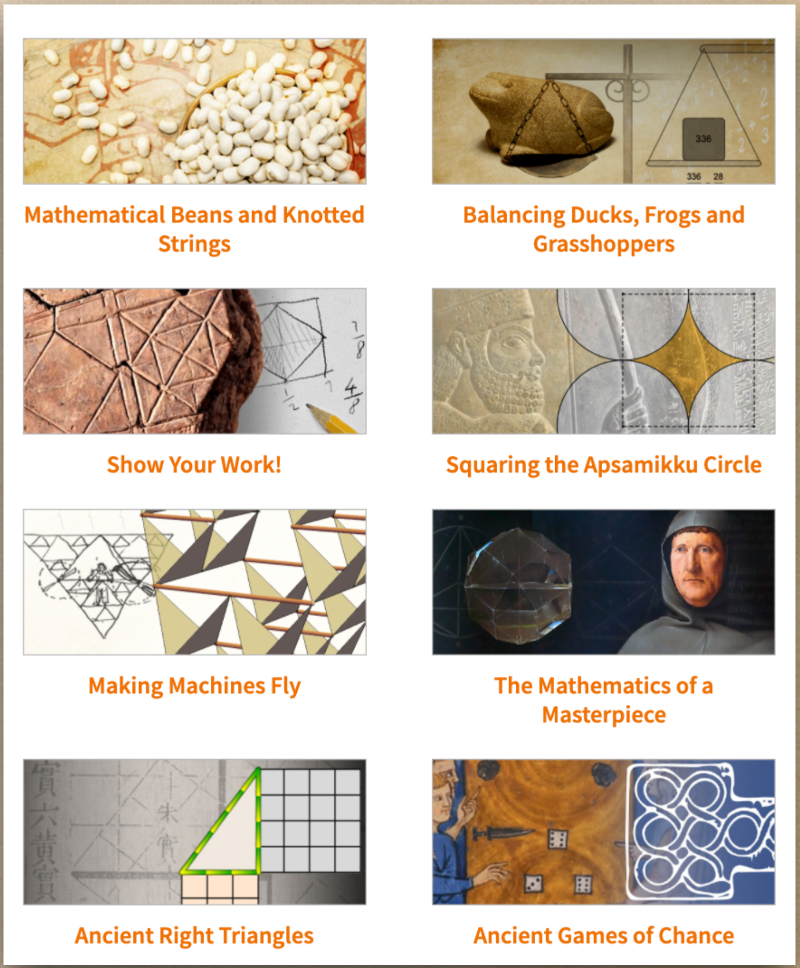
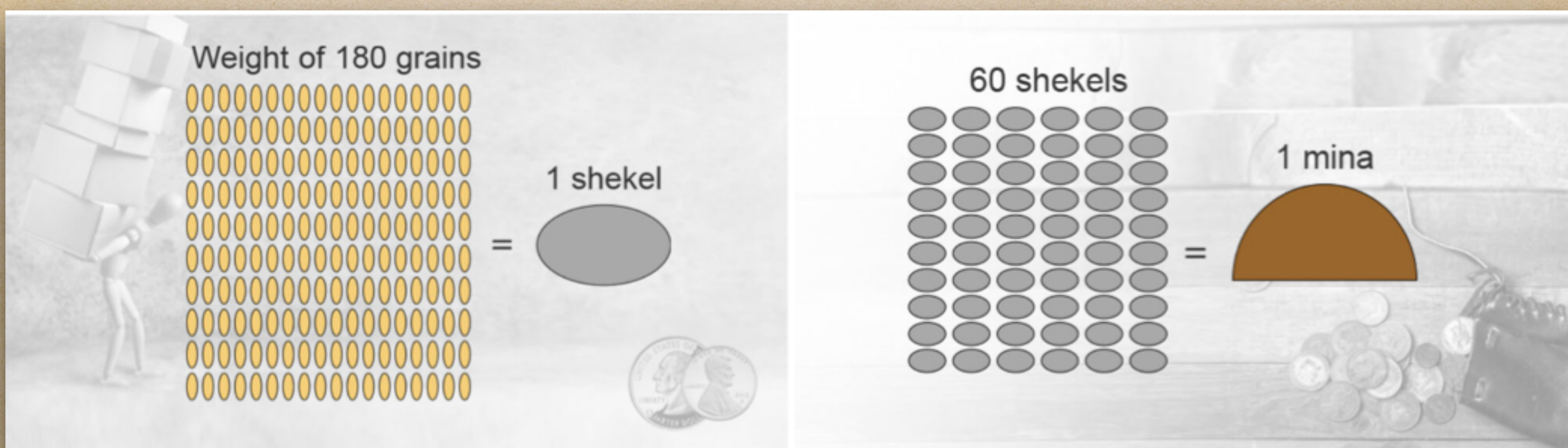


Balancing Ducks, Frogs and Grasshoppers

Barley was so important to the ancient Mesopotamians that a barley grain was used as the smallest unit of length, area, volume and weight. A shekel of silver weighed as much as 180 barley grains, or about 8.4 grams. 60 shekels weighed 1 mina, and 60 mina weighed 1 talent.



Balancing Ducks, Frogs and Grasshoppers

Merchants would carry around their own set of weights to help them with trading. Most weights were sort of grain-shaped. Mesopotamian weights were often made of polished hematite:

Hematite weights ranging from three shekels to one mina. Uruk, Mesopotamia, ca. 2000–1600 BCE.

Mesopotamian weights were often shaped like a sleeping duck, with its neck and head resting on its back:

Left: Duck-shaped hematite weights, Mesopotamia, ca. 2000 BCE.
Right: A sleeping duck!

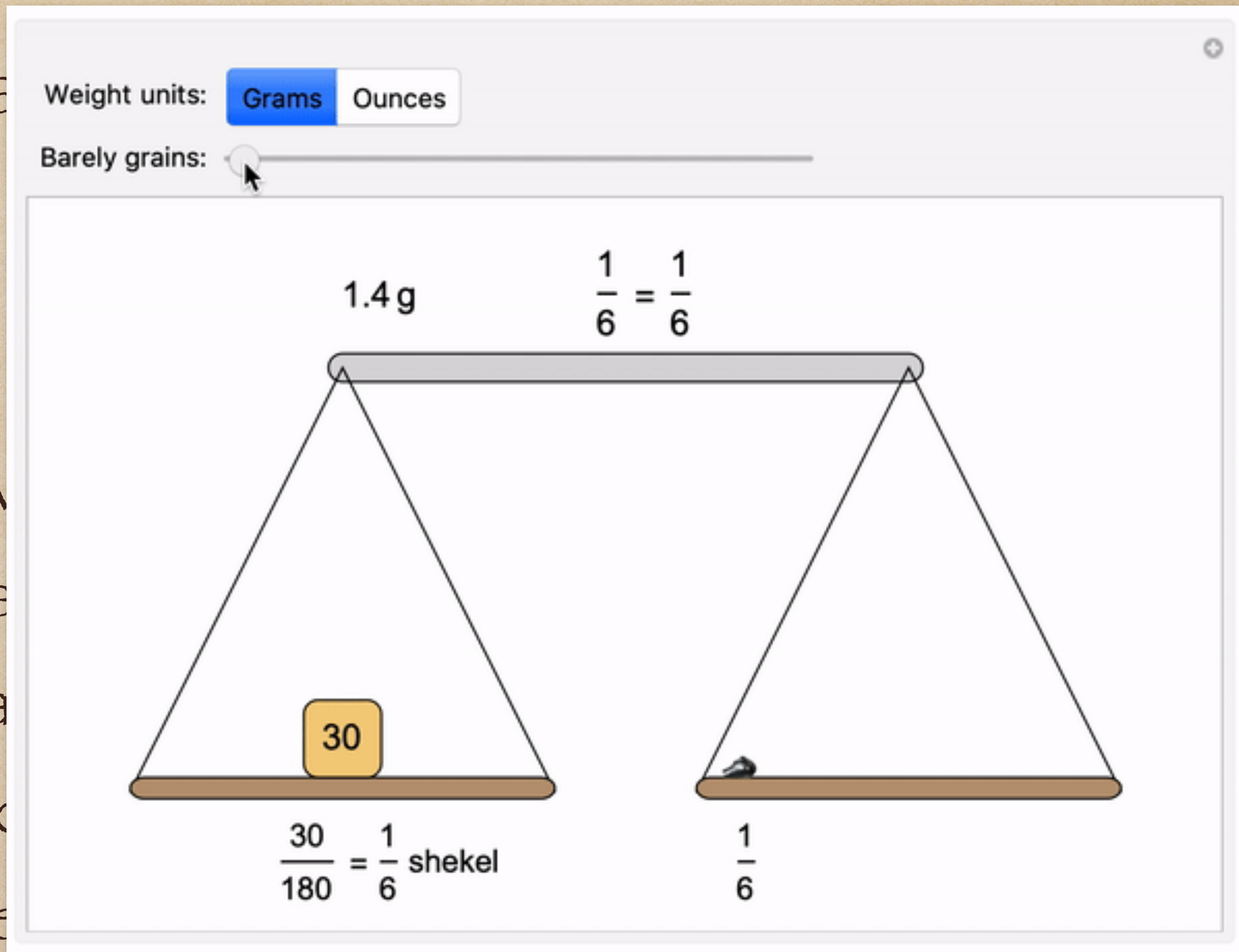
There are rare examples of Mesopotamian weights in other shapes, but most weights were either grain-shaped or duck-shaped. Here are some unusual examples of Mesopotamian weights: a grasshopper, a shell and a cute frog:

Left: Mesopotamian grasshopper weight made of hematite, ca. 1800–1600 BCE.
Center: Mesopotamian shell weight made of hematite, ca. 1800–1600 BCE.
Right: Mesopotamian frog weight, ca. 2000–1600 BCE. The Akkadian inscription under the frog's throat reads: "a frog [weighing] 10 mina, a legitimate weight of the god Shamash, belonging to Iddin-Nergal, son of Arkat-ili-damqa."

Balancing Ducks, Frogs and Grasshoppers

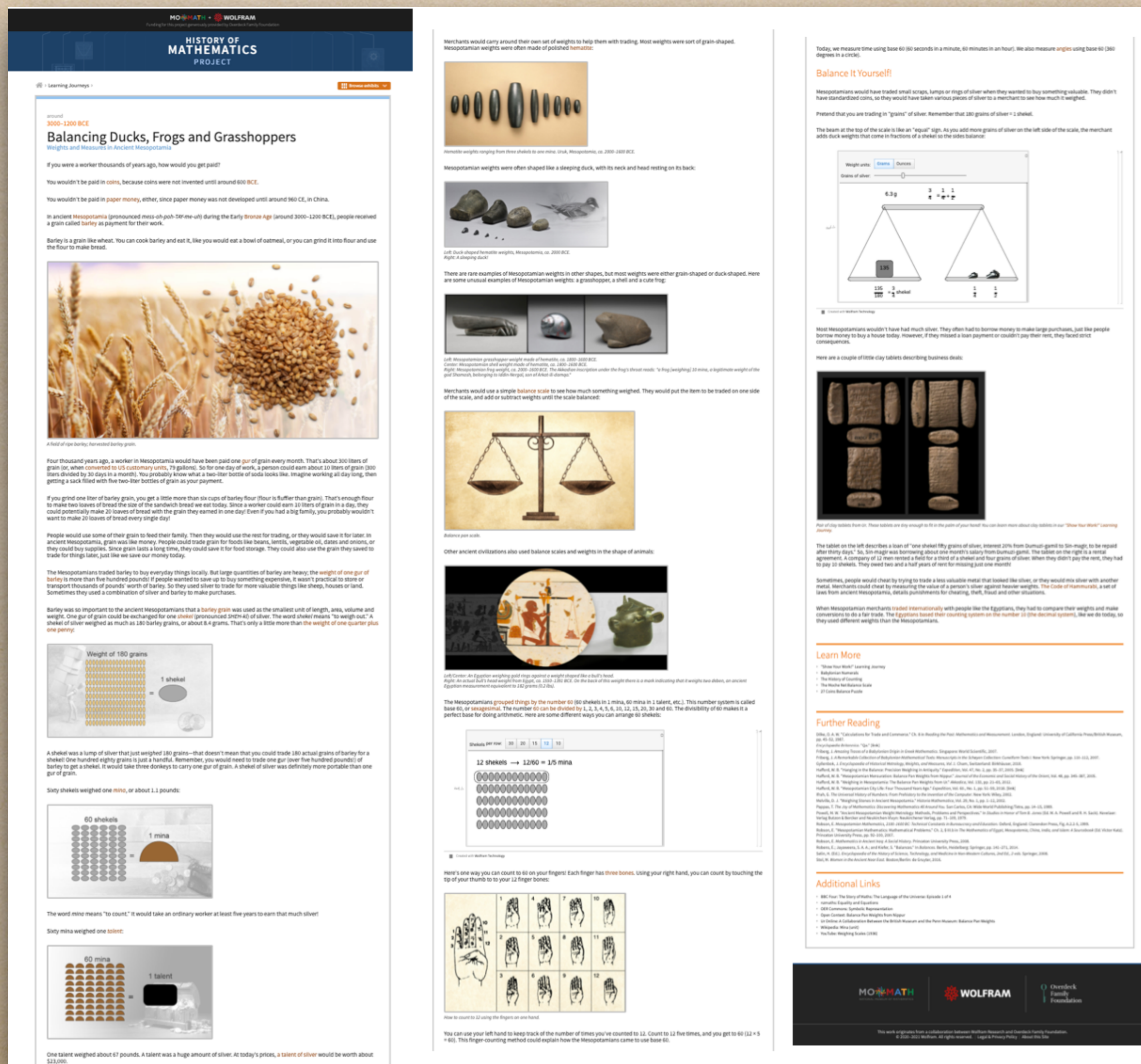
- To understand how these weight stones might have been used I created the following balance scale interactive. As you add more barley grains on the left side of the scale, the merchant adds duck weights that come in fractions of a shekel so the sides balance. The beam at the top of the scale acts as an "equal" sign!

Bala ... bers



- To ... ght hav ... nce sca ... gra ... chant add ... f a she ... he top of the scale acts as an "equal" sign!

Balancing Ducks, Frogs and Grasshoppers



Benefits of a Wolfram Language build system

- Easy to curate all relevant data (text, images, metadata, mathematical and interactive content) in a single place: notebooks
- Content elements such as maps, timelines, and thumbnails can be generated completely programmatically using built-in Wolfram Language functionality
- Incremental builds are easy simply by checking for changed notebook content

Thanks!

- Overdeck Family Foundation
- MoMath, the Museum of Mathematics in New York City.
- Stephen Wolfram
- Andrea Gerlach, Eric Weisstein, Sarah Keim Williams
- Lori Goodman, Sushma Kini (project management)
- Michael Trott (content suggestions and review), Christopher Wolfram (content suggestions and discussions), Dan McDonald (synthetic geometry contributions), MinHsuan Peng (custom timelines), Shadi Ashnai and Giulio Alessandrini (image processing)
- Heidi Kellner and Jeremy Davis (web design), Marion Morris (web implementation), Taylor Birch (proofreading)
- Our network of 50+ domain and content experts

Background and Timeline

- In 2019, Stephen Wolfram proposed a project to develop a virtual interactive collection of mathematical artifacts for the Museum of Mathematics (MoMath) in New York City
- The project was generously funded by Overdeck Family Foundation
- Over the last two years, researchers at Wolfram Research have investigated and written up detailed histories, descriptions, and explanations for a collection of mathematical artifacts
- The results have been incorporated into a website (history-of-mathematics.org) created using a custom build system modeled after the one being used for Stephen Wolfram's Physics Project

Build system

- Website is built using the Wolfram Language
- Source documents are tagged notebooks [example]
- All content built to and hosted in the Wolfram Cloud
- Computational/interactive content are simply notebook sections embedded directly in the cloud using WolframNotebookEmbedder
- Core workflow based on XMLTemplate + ExportForm:

Build system

- Website is built using the Wolfram Language
- Source documents are tagged notebooks [example]

```
CloudDeploy[
  ExportForm[
    XMLTemplate[File[CloudObject[$HTMLTemplateFile]]][xmlfile],
    {"Text", "HTML"}
  ],
  $MathArtifactsURLBase <> url,
  Permissions → "Public"]
```

- Core workflow based on XMLTemplate + ExportForm:

8 Learning Journeys

- <u>Mathematical Beans and Knotted Strings</u>. Counting Methods from the Moche Culture.
- <u>Balancing Ducks, Frogs and Grasshoppers</u>. Weights and Measures in Ancient Mesopotamia.
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